A boy or a girl?

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Imagine, Dear Reader, that you are embarking on a long train journey. Just as you have comfortably settled in your seat and started reading your favourite magazine, your are interrupted by a cheeky exclamation: “Hey, what a coincidence! Long time no see, huh?” You raise your head, and your eyes meet a friendly face that, with a little effort of memory, you recognize as an old friend from elementary school.

“Yeah, long time indeed. . . ” you reply, reluctantly closing your *Delta* magazine. “How have you been?” your friend courteously inquires.

Well, there comes a moment when you have to summarize several years of life in one sentence, so you respond, “Good! And how about you?” Your friend opens his mouth, and you already know that the question was a mistake. He **really** wants to summarize the past few years of his life, certainly not in one sentence. After an hour of narrating his school and professional adventures, the conversation shifts to family matters. At some point, you limit yourself to nodding politely, smiling, and the infallible “Oh boy!” while in your mind you are actually solving the Delta’s Problem Corner [in this issue on page 19, ed. note].

At some point, you realize that your friend has started talking about the difficulties of finding childcare for his child. The problem is that you missed the moment when he mentioned having a child at all.

Hmm, you think to yourself, *I didn’t hear whether it’s a boy or a girl. It’s better to refer to them in a gender-neutral way; otherwise, I have about a 50% chance of an awkward mistake.*

A similar issue is addressed in the song *Nie mam pojęcia* by Łona and Webber.

As the conversation continues, it turns out that your friend was seeking childcare not for one child but for two! Furthermore, you caught the sentence in his monologue, “I went for a walk with my son,” which establishes the gender of one of the children. The gender of the second child remains a mystery to you.

Ah, I once read something about this on the internet! you think. *At first glance, it would seem that the chance of the second child also being a boy is 50%. But we can look at it differently. If I forget the information I already have, the chance of my friend’s older child being a boy is 50%, and similarly, the chance of the younger one being a boy is also 50%. Therefore, the chance of having two boys is 50% × 50% = 25%. Likewise, the chance of having two girls is 25%. There are two remaining possibilities, each with a probability of 25%: older boy, younger girl, and vice versa. I already know that my friend has a son, so out of these four equally likely situations, I can restrict attention to three (by excluding two daughters). And out of those three, only one corresponds to the situation of my friend having two sons, so the chance of that is 1/3. With my back against the wall, I should bet on a daughter!*

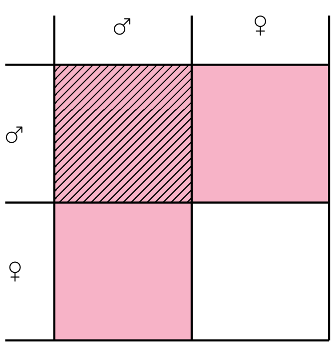

Fig. 1. The rows correspond to the gender of the older child, and the columns correspond to the gender of the younger child. The hatched area represents 1/3 of the coloured region

To ensure the correctness of your reasoning, you discreetly took out a piece of paper and a pencil and sketched a convincing 2 × 2 table representing different combinations of the gender of the older and younger child (reproduced in Figure 1). Yes, it clearly shows that everything is correc. . .

(voiceover) No! Nothing is correct! As a matter of fact you are dealing with a situation where a randomly encountered person (the information that it is your acquaintance is irrelevant) happens to have two children and starts telling you about one of them, who turns out to be a boy. It is reasonable to assume that your acquaintance randomly selected one of his children (each with equal probability). If he is the father of two boys, he will definitely talk about a boy. If he is the father of a boy and a girl, he has a 50% chance of talking about a boy. In other words, out of all fathers of two children of both genders, only half of them would start talking to you about their son. Thus, your table should correspond to the one in Figure 2. In the end, from your perspective, the chance of your acquaintance having two sons remains 50%!

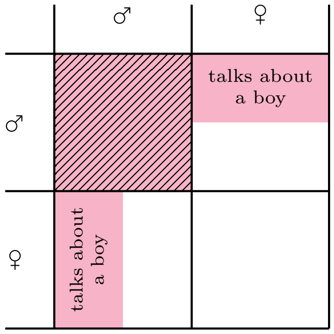

Fig. 2. The hatched area represents $1/2$ of the coloured region

However, it would be a different scenario if you specifically asked your acquaintance, “Dear friend, please start telling me about one of your sons if you have at least one.” Assuming that your acquaintance would indeed begin a story (instead of quickly changing seats), the reasoning and table presented earlier in Figure 1 are correct, and the chance of him having two sons would indeed be $1/3$.

For the purpose of the rest of the story, let’s assume that was the case.

The monologue of your acquaintance continues. Once again, you concentrate on something else. At some point, a piece of information breaks through your consciousness: the aforementioned son was born on a Monday, and since then, Monday has been your acquaintance’s favorite day of the week (lucky him). This news prompts you to engage in another mathematical reflection.

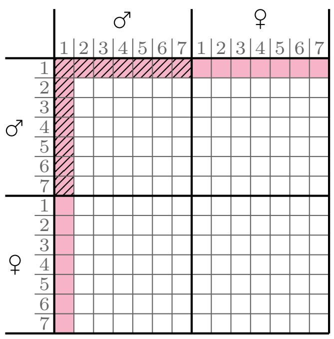

Fig. 3. The numbers correspond to the day of the week on which each child was born. The hatched area represents $13/27$ of the coloured region.

Should the information I just obtained change my estimation of the chances that my acquaintance has two sons? It might seem that it shouldn’t in any case since the day of the week on which his mentioned son was born cannot influence the gender of the other child in any way. However, on the other hand, this knowledge requires me to modify the previously sketched table. Now, for each child, the older and the younger, I should record not only their gender but also the day of the week on which they were born. Assuming that each day of the week is an equally likely birthday, this gives me 14 equally likely configurations of gender and day of the week for each child, resulting in a total of 196 configurations for both children ($14^2 = 196$)*. This time, 27 of them correspond to the occurrence of a boy born on Monday, and out of those, in 13 cases, the other child is also a boy. Therefore, my estimation of the probability that my acquaintance has two boys should increase from* $1/3$ *to* $13/27$.

Because the situation intrigued you, you redrew your table, obtaining something similar to Figure 3 in the margin. And although you know from experience that tables are not to be debated, the conclusions you reached still bothered you.

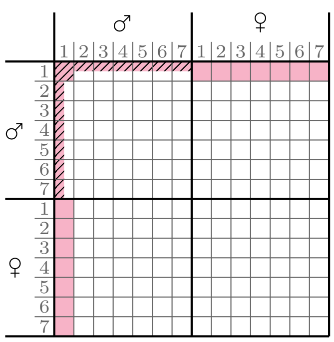

Fig. 4. The hatched area represents $1/3$ of the coloured region

(voiceover) And rightly so, as such reasoning replicates the mistake described earlier. This time, you should take into account the fact that if your acquaintance has two boys, with exactly one of them being born on Monday, he started talking about that specific child, but he could have talked about “the other one”, which effectively reduces the probability of such a configuration from your perspective. Considering this in the reasoning leads us back to the answer of $1/3$ (as before), as can be seen by analyzing Figure 4. Again, if **you** were to ask whether your acquaintance has a son born on Monday, an affirmative answer would allow you to change the estimation of the chances of having two boys to $13/27$, in line with the reasoning presented earlier (while a negative answer would decrease the estimation to exactly 30%, as we encourage you to verify independently).

After this information, the topic of children came to an end. Soon, after listening to a few more gripping stories from your acquaintance’s life, the train arrived at its destination – it did not surprise you when it turned out that both of you were getting off at the same station. On the platform, his wife was already waiting with a tousled rascal.

Question 1. How do you now assess the chances that your acquaintance has two sons?

As you were about to say your goodbyes, your old preschool friend, whom you haven’t seen in ages, appeared behind your school acquaintance! Before she could say anything, you shouted to her:

“Hey there, long time no see! Do you happen to have two children, at least one of whom is a boy born on Monday?”

Your friend was taken aback for a moment, then smiled cunningly and replied:

“Yes, I have two children, and none of them was born on Monday.”

Question 2. What are the chances that your preschool friend has two sons?

(Answers to the above questions can be found on page 14.)